DOCUMENT RESUME

ED 341 549 SE 052 426

AUTHOR Remillard, Janine
TITLE Abdicating Authority for Knowing: A Teacher's Use of an Innovative Mathematics Curriculum. Elementary Subjects Center Series, No. 42.
INSTITUTION Center for the Learning and Teaching of Elementary Subjects, East Lansing, MI.
SPONS AGENCY Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE Jul 91
NOTE 36p.
AVAILABLE FROM Institute for Research on Teaching, 252 Erickson Hall, Michigan University, East Lansing, MI ($3.25).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Case Studies; *Curriculum Development; Educational Change; Elementary Education; Mathematics Education; Mathematics Instruction; Teacher Attitudes; *Teacher Behavior; Teacher Student Relationship; *Teaching Methods
IDENTIFIERS *Comprehensive School Mathematics Program; Mathematical Thinking; Subject Content Knowledge

ABSTRACT

This paper describes the case of Cassandra Singer, a third-grade teacher finishing her first year of using Comprehensive School Mathematics Program (CSMP), an innovative mathematics curriculum. Drawing from interviews and observations, the document explores relationships between Cassandra's knowledge and beliefs about mathematics, how it is taught and learned, and how she transforms and is transformed by the text. Relinquishing her authority for knowing to the text, Cassandra aims to follow the lesson scripts provided in the manual, while interpreting their purpose through the lens of her own beliefs. An analysis of the mixture of procedurally and conceptually oriented practice observed in this classroom in light of the teacher's beliefs and their relationship to the CSMP curriculum is presented. This case raises questions about the balance of authority between the curriculum and the teacher and students. Concluded is that although CSMP is a carefully designed curriculum that can potentially revolutionize teaching, the teacher needs the subject-matter knowledge to avoid a procedural use of the curriculum. (Author/MDH)

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ED341549

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ABDICATING AUTHORITY FOR KNOWING: A TEACHER'S USE OF AN INNOVATIVE MATHEMATICS CURRICULUM

Janine Remillard

Center for the Learning and Teaching of Elementary Subjects

Institute for Research on Teaching
College of Education
Michigan State University

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Janine Remillard

Published by

The Center for the Learning and Teaching of Elementary Subjects
Institute for Research on Teaching
252 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

July 1991

This work is sponsored in part by the Center for the Learning and Teaching of Elementary Subjects, Institute for Research on Teaching, Michigan State University. The Center for the Learning and Teaching of Elementary Subjects is funded primarily by the Office of Educational Research and Improvement, U.S. Department of Education. The opinions expressed in this publication do not necessarily reflect the position, policy, or endorsement of the Office or Department (Cooperative Agreement No. G0098C0226).

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The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

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Abstract

In this paper the author describes the case of Casandra Singer, a third-grade teacher finishing her first year of using Comprehensive School Mathematics Program (CSMP), an innovative mathematics curriculum. Drawing from interviews and observations, the author explores relationships between Casandra's knowledge and beliefs about mathematics and how it is taught and learned and how she transforms and is transformed by the text. Relinquishing her authority for knowing to the text, Casandra aims to follow the lesson scripts provided in the manual, yet she interprets their purpose through the lens of her own beliefs. The author presents an analysis of the mixture of procedurally and conceptually oriented practice observed in this classroom in light of the teacher's beliefs and their relationship to the CSMP curriculum.

ABDICATING AUTHORITY FOR KNOWING: A TEACHER'S USE OF AN INNOVATIVE MATHEMATICS CURRICULUM[1]

Janine Remillard[2]

In this study I consider how a teacher uses an alternative mathematics curriculum, which proposes a view of mathematics grounded in the discipline. This study links research on the relationships between practice with three significant domains--teacher beliefs and reasoning, reform in mathematics education, and the role of curriculum in reform--by examining how a teacher's knowledge and beliefs about mathematics teaching and learning shape how she interprets and uses the innovative curriculum. The ground work for this study has been laid by research establishing connections between what teachers know and believe and how they use curriculum. In light of current interest in changing how and what mathematics is taught, I look at these connections in a case in which the published curriculum is one that represents mathematics as a unified set of ideas and a way of thinking, rather than a collection of disconnected rules to be memorized. This perspective challenges traditional notions about teaching and learning mathematics but is in accord with current calls for change in mathematics education. My aim is to consider issues relevant to supporting teachers who are making changes in their mathematics teaching, particularly the role of published curricula in such endeavors.

The Relationship Between Teacher Beliefs and the Practice of Teaching

What teachers know and believe about the content they are teaching and how it is taught and learned are powerful determinants in what is taught of and about the content (Ball, 1988). These teachers' beliefs are influenced significantly by their experiences as students and

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[1]Paper originally presented at the annual meeting of the American Educational Research Association, Chicago, IL.

[2]Janine Remillard, a doctoral candidate in teacher education at Michigan State University, is a research assistant with the Center for the Learning and Teaching of Elementary Subjects. The author gratefully acknowledges Casandra Singer and her third-grade students for their participation in this study; Margery Osborne, Nancy Jennings, Penelope Peterson, and Linda Anderson for their helpful feedback on earlier drafts of this paper; and Deborah Ball for her unending support.

as learners of the particular subject (Buchmann, 1987). Sixteen years as a student are powerful determinants in teacher beliefs and actions; thus, teachers tend to teach the way they were taught and bring these ideals, learned through an apprenticeship of observation (Lortie, 1976), into their own classroom.

Teacher beliefs about the content they teach are also shaped by popular conceptions of that subject. This is particularly evident in mathematics. As Ball (1988) points out, teachers' and students' beliefs about mathematics are also heavily influenced by those of the mainstream culture, particularly the belief that it is a cold, hard, mysterious domain. This conception of mathematics has grown out of how mathematics is traditionally taught in schools. Teachers generally represent the substance and nature of the school subject of mathematics as a fixed body of knowledge made up of little bits to be memorized and recalled quickly. While recognizing that this "enacted curriculum" in mathematics teaching "is itself a representation of the discipline" (p. 303), Ball argues that it represents a view of mathematics that is antithetical to the discipline.

Thompson's (1984) study, which relates teachers' conceptions of mathematics to their instructional practices, suggests that teachers' beliefs, views, and preferences about mathematics and its teaching play a significant role in shaping their teaching of mathematics. The teachers who viewed mathematics as a static body of knowledge presented the content as a finished product--rules and procedures to be learned or memorized. Nevertheless, their approaches varied according to their views of the interrelationship between topics and how students learn. The teacher who held a dynamic view of mathematics engaged her students in a "creative generative process" (p. 119). In more recent studies, Schoenfeld (1989) found contradictions between teachers' and students' professed beliefs about mathematics and how it is represented in the classroom. He ascribes this to a "rhetoric of mathematical understanding" (p. 344), a product of the language of current reform efforts in mathematics, which has influenced how mathematics is talked about but has not penetrated deeply enough to affect instructional practices.

Recent research also suggests that teachers' knowledge, beliefs, experiences, and perceptions of outside pressures shape what is taught in the classroom. Findings of the Content Determinants Project, which looked at teacher control over content taught in mathematics classrooms, suggest that teachers' decisions which ultimately determine content are shaped by their perceptions of pressures and expectations brought on by testing, parents, or district policy (Floden, Porter, Schmidt, Freeman, & Schwille, 1981); commitments to the content, attitudes toward teaching certain areas, their assessment of their own competence in teaching these areas (Buchmann & Schmidt, 1981); and their level of mathematical expertise (Kuhs & Freeman, 1979). Thus, the teacher's perceptions of contextual concerns, added to her beliefs about the subject and her learners, influence what is taught.

In contrast, there is some evidence that teachers are also heavily influenced by textbooks or written curricula (Freeman & Porter, 1989), and textbooks are prevalent throughout all domains of teaching (Squire, 1988). According to Komoski (1985), 90% to 95% of classroom time is spent using commercially prepared classroom materials. What role do these curriculum materials have in determining what is taught in the classroom?

The Relationship Between Curriculum and Practice

Research on teaching suggests that each teacher interprets and comes to understand a text individually, through the unique lens of his or her own experience, knowledge, and beliefs. What one sees in the classroom is a teacher's interpretation of the textbook authors' intentions. Graybeal and Stodolsky's (1987) study of commonly used mathematics and social studies texts includes analyses of the texts, followed by observation of teachers using them. Their findings suggest that the alterations teachers make to published curricula tend to lean more toward traditional conceptions of mathematics--focusing more on rote memorization of isolated facts than on understanding concepts and application--than was intended by the text authors.

Studies in other subject areas have explicitly examined relationships between the teachers' knowledge and beliefs <u>about</u> the subject and their role in teacher interpretations and transformation of textbooks. FitzGerald's (1979) research suggests that a teacher's conception

of what social studies contains influences how a text is used and thus how social studies is represented in the classroom. Durkin (1983) links teachers' views of reading with their undertreatment of comprehension and overtreatment of skill development, contrary to suggestions made in teachers' manuals of basal readers. Case studies of science teachers' use of science textbooks reveal that teachers and curriculum developers hold different views about learning and the nature of science which influence how teachers guide their students through the material in the science textbooks. As a result, the teachers studied were unsuccessful at helping students replace misconceptions with scientific conceptions held by the teacher and presented in the text (Roth & Anderson, 1987; Smith & Anderson, 1984).

Findings of a current policy into practice study of the California State Mathematics Framework suggest that the relationship between teachers' views of mathematics is not necessarily unidirectional. This study found evidence of teachers transforming texts and texts transforming teachers. Examples were found of omissions and alterations made by teachers of pieces of textbook lessons. At the same time the case studies reveal ways in which textbook changes have had altering effects on teachers' practices and consequently on some of their beliefs about mathematics (Remillard, 1990b; Wiemers, 1990). For many of the teachers studied, certain elements of their practices were more susceptible to change than others (Ball, 1990). The teachers' conceptions and knowledge of mathematics influence their selective integration of the state's reform initiative into their practice. In one sense, these teachers were "following" their textbooks; but in another sense, what they were actually following was a perspective of the curriculum which was constructed by their beliefs.

The Relationship Between Beliefs and the Implementation of an Alternative Curriculum

This study considers what happens when a teacher, Casandra Singer,[3] uses a curriculum which proposes a completely new way of thinking about teaching mathematics--a perspective that is grounded in the discipline of mathematics. The Comprehensive School

[3]Casandra Singer, the names of her students, and the name of the school are pseudonyms.

Mathematics Program (CSMP)[4] is an example of a mathematics curriculum designed to bring mathematics that grows out of the discipline into the elementary classroom. Emphasizing "mathematically important ideas" and reasoning, the CSMP authors hold that "elementary school mathematics should not unduly stress drill and practice in arithmetic computation" (Herbert, 1984, p. 3) but should present mathematics as interrelated, complex, and interesting ideas which children can reason about and use to solve problems. In addition to presenting an alternative view of mathematics content, CSMP also proposes that learning mathematics occurs through engaging in mathematical thinking, developing strategies to solve problems, and communicating about mathematical ideas within the context of whole-class lessons.

Rather than focusing on algorithmic operations, CSMP lessons give much greater emphasis to understanding underlying mathematical concepts. Three visual, representational languages (pictorial representations) are employed consistently throughout the program by the authors to model mathematical ideas and provide young students with access to abstract mathematical concepts. Also, the lesson suggestions are presented in a scripted dialogue, to guide the teacher through each lesson. The interwoven, spiral organization of the content, the use of these representational languages, and the emphasis on whole-class discourse suggest epistemological and pedagogical changes to traditionally held views of mathematics.

CSMP presents a view of mathematics and a way of teaching it which is very uncommon to the elementary curriculum and to the way the majority of teachers experienced mathematics learning. Unlike the minor and perhaps unconnected changes to the school subject of mathematics found in the commonly used texts in California, CSMP proposes major changes in content and pedagogy which suggest a closer match between the school subject and the discipline of mathematics. This adds complexity to the teacher's process of interpreting such a

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[4]The Comprehensive School Mathematics Program is published by the Mid-Continent Regional Educational Laboratory, Kansas City, MO, 1978. See Remillard (1990a, 1991) for more detailed descriptions of CSMP.

text. In other words, in using CSMP, teachers are being asked to teach in a way they have never experienced before as a teacher or learner (Cohen, 1988).

This study looks closely at one teacher's, Casandra Singer's, interpretation and understanding of CSMP as she uses it in her third-grade classroom, uncovering links between her knowledge and beliefs and her use of the program. A close look at how a teacher makes sense of an innovative curriculum can inform current efforts in reforming mathematics education. For, in order to effect change in how and what mathematics is taught, reformers must consider how teachers might learn to teach mathematics in ways they have never experienced.

Data Collection and Analysis

Data Collection

In order to understand what influenced her interpretation and use of CSMP, I collected data on her knowledge and beliefs about mathematics, teaching and learning, and on her actual use of CSMP. The data collection was in three forms: a baseline interview, classroom observations, and postobservation interviews.

Baseline interview. This extensive interview provided background data on Casandra's knowledge and beliefs. Intended to develop a context in which to situate my observations, the interview was conducted in two parts before the classroom observations began. Its purpose was to collect background information on the teacher, including her experiences teaching and learning mathematics, her current beliefs about and knowledge of and about mathematics, and her views on CSMP.

Classroom observations. I observed six consecutive lessons of Casandra's math class over a period of two weeks. Each observation included a brief preobservation interview, during which time she prepared me to observe the lesson by explaining what she planned to do, how she hoped it would go, and what she was hoping to accomplish.

The observations included all the activities taking place during the math period. All but one of these periods included whole-class instruction. The focus of the observations was

the teacher's use of CSMP and how mathematics was construed in the class--how the substance and nature of mathematics was being represented. I audiotaped each lesson and took detailed field notes on the established patterns or routines in the class, such as how Casandra asked questions and the kinds of questions she asked, the mathematical tasks the students engaged in and how they were framed and pursued, how she responded to her students and how they responded to her, as well as the type of discourse which took place and how it was orchestrated. Through the observations of such events I looked for patterns in the teacher's and students' actions that suggested certain beliefs and attitudes about mathematics: what it means to know it, what the subject contains, and its relevance in their lives.

Following each lesson, this observational data was used to write an analyzed account of the lesson using the observation instrument developed by the California Policy Into Practice Study. In addition to beginning with a narrative account of the lesson, the instrument framed questions about the lesson's content, organization, structure, discourse, and instructional representations

Postobservation interview. I also conducted three postobservation interviews with the teacher which focused on her thinking about and reaction to the lessons I had observed.

Data Analysis

Interview data. The following conceptual categories were used to analyze the interview transcripts: beliefs about mathematics, learning and teaching, learners, the context, and CSMP. These provided a balanced and well-developed theoretical frame for analyzing pedagogical reasoning (Ball, 1988). Beginning with the interview data, I developed a description of Casandra's beliefs under each category and then looked for connections between these beliefs. After developing assertions about her beliefs, I turned to the observational data for confirming and disconfirming evidence.

Observational data. Looking at the places that Casandra made modifications in the CSMP script, I used the postobservation interviews to assess the possible underlying purpose and intent of each modification. Believing also that the parts of CSMP that she chose not to

modify were equally significant in understanding her use of the curriculum, I made note of these points and then looked for patterns in mathematical content, representations, lesson content, and classroom atmosphere within these categories. The interview data were used to bring meaning to the patterns that emerged in her moves from and adherence to the script in the teacher's guide.

Developing a case. Through the data analysis, patterns emerged in her teaching: Her practice seemed to present a mixture of procedurally and conceptually oriented views of mathematics. This varied according to the content of the lesson, how the students were responding, and whether or not she was following the script. She seemed not to engage intellectually the mathematical ideas in the lesson, leaving that to the manual, but focused on moving the lesson along according to the script in the manual or in her head. I have, thus, chosen to present Casandra as a case of a teacher whose relinquishing her authority for knowing to the program resulted in a procedural and conceptual mix. Using this perspective, I have tried to understand what might be leading her to give up her authority in this way and to examine what happens in her class as a result.

The case of Casandra Singer, however, is reported from only one perspective. Other cases could be defensibly written from other vantage points. There is a lot to learn about Casandra from this case, or others that could be written. But there is also a lot to learn about teaching and using curriculum that seems relevant to people interested in changing mathematics practice. The particulars of this case cannot be generalized to other teachers, but the issues do transcend this one situation. Indeed, Casandra Singer, in one sense, is every teacher. What we take from such a perspective on teaching is solely determined by how we understand this teacher and her practice.

Abdicating Authority for Knowing: The Case of Casandra Singer

> Let's face it, elementary teachers are not experts in everything; we need the help of [pause]. There's too much knowledge out there. I need experts to tell me, to design things so that I can be effective. We don't know everything. We have our weaknesses and our plusses. And even with the plusses, you don't

> know everything about it. There's too much and there's too much new stuff out there. . . . I would feel more comfortable having [the math curriculum] written by mathematicians, having it designed so kids can learn it than to have publishers getting in on something that's just going to sell. (Interview, 4/90)

Casandra Singer is not satisfied that her students have strong computational skills, that they know their times tables when they come to her in third grade or that they can get the right answer. These things are important to her, but she wants them to understand and explain why their answers are right, to use math to solve problems in their lives, and to enjoy it. To her pleasure, however, she has found CSMP, an elementary math program that she feels can potentially accomplish the goals she has for her students. It was adopted by the district because of its attention to developing conceptual understanding and mathematical thinking. Although it was adopted as her district's core mathematics curriculum several years ago, the implementation has been gradual. Thus, this is the first year Casandra has used it.

Valleyrock, the school where Casandra teaches third grade, is located in a small, but visible, suburban district. Like many high-achieving, affluent school systems caught in the intersection of parental demands and educational pressures to keep up with current innovations, the district, and Valleyrock in particular, face the formidable challenge of maintaining high test scores while responding to calls for innovation and reform. So it was somewhat controversial when, in the face of already high standardized test scores but charges of low conceptual understanding, the district adopted a nontraditional mathematics program--one that claims that traditional algorithms should be taught well after the related concept has been introduced and developed (Herbert, 1984). To assist teachers in using such an unusual program, the district offered inservice training. Also, the CSMP manual contains lesson scripts, in a question and answer format, that the teacher may follow or use as a guide when teaching.

Casandra's teaching and inservice experience has convinced her that the abilities and dispositions she wants her students to develop are determined by how they are taught. In fact, she believes that if she had been taught math using CSMP, instead of by memorizing and then regurgitating meaningless rules and formulas, she, and probably the majority of the adult population, would feel more comfortable with math. "I know now that if a lot of us had been

taught this way, maybe we would have developed an interest. Maybe math would have meant something different than what it does now" (Interview, 4/90). But she was not taught in this way and is uncomfortable with math. So she relies on the CSMP authors to have designed an "effective program." Casandra feels confident that, through the use of CSMP, she will avoid subjecting her students to an experience that replicates her own.

To Casandra, using CSMP means sticking to the written curriculum, following closely the scripted lessons. It would seem, then, that her 25 students are receiving mathematics instruction that is just as the CSMP authors suggest. But like all words, spoken or written, the text of the CSMP curriculum is being interpreted by Casandra. As she uses it she transforms it, through the lens of what she knows and believes, from written text to something alive in the classroom. This transformation is evident not only in what she includes and omits, modifies, and embellishes but also in the role it plays in her complete math instruction and in her instruction across the school day. For example, Casandra's choices to relegate CSMP to three afternoons a week, designate the mornings for reading instruction, and use a traditional textbook (Addison-Wesley Mathematics, Menlo Park, CA, 1987) to supplement CSMP's offerings with computational practice, is relevant in understanding her interpretation of CSMP and the purpose of mathematics instruction in the context of the entire school curriculum. In other words, her view of what it means to know mathematics and what knowing mathematics means is reflected, to some degree, in how she uses the curriculum. Yet, even this relationship is distorted by the pressure of multiple demands teachers face.

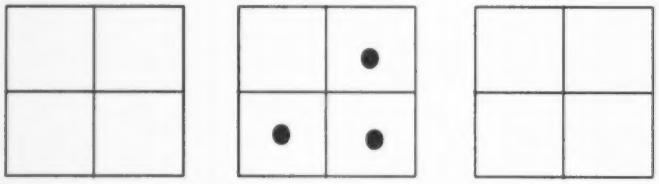
Thus, in order to capture and understand Casandra's mathematics teaching we must look past the written text to her actual treatment of it in her teaching. Following the CSMP manual, Casandra presents tasks to her students that are designed to focus on underlying concepts. But what she makes central in the lesson is often a set of convergent responses or mechanical steps of working with the representation. In one lesson, for example, the class was

using the minicomputer[5] to show that half of 70 dollars is 35 dollars. Casandra had posed the situation as follows:

Casandra: Carl and Conner are my friends. I want to share 70 dollars between them. How much will each get?

Laura: Thirty-five dollars.

Following the script, Casandra asked, "Is there a way we can show this on the minicomputer? Who can come up and put 70 on our minicomputer?"[6] Rosa went to the board and put the following configuration on the minicomputer:



[5]The minicomputer is one of the three visual representations used in CSMP. One of its strengths is its representation base-10 place value. There is one board for each place: ones, tens, hundreds, and so forth. Numbers are shown on the board by placing magnetic checkers on the individual colored squares (which have the values shown below) and summing their value.

8	4
2	1

For example, a checker in the 2 square and one on the 4 square show 6. The same configuration on the ten's board shows 60. Computations are done on the boards by "making plays," which means moving and exchanging checkers while keeping the value of the number the same. Two checkers on the 4 square can be exchanged for 1 checker on the 8 square, as two 4s are the same as 8. To make a play from the ones board to the tens board a checker the 2 square and the 8 square are moved to the 10 square on the next board, as 8 and 2 is the same as 10. Backward plays are similar. Since 40 is the same as two 20s, a checker on the 40 square can be exchanged for two checkers on the 20 square. To take half of a number, backwards plays must be made until there are exactly two checkers on each square. Then half of the checkers, one from each square, are removed.

[6]It may seem like a waste of time to go through the steps of taking half of 35 on the minicomputer since the class already knew the answer, but the authors recommend that the class actually make the plays on the minicomputer to remove half of the checkers to visually illustrate taking half of a number. Doing this highlights the concept of halving and reviewing the process of doing this on the minicomputer, thus preparing them for the next part of the lesson: taking half of seven, which results in nonwhole number.

(Rosa has shown 70 by placing checkers in the four, two, and one squares on the board in the ten's place. Her representations shows that 40 + 20 + 10 = 70.)

Casandra: Okay, how can we show half of 70 now on the minicomputer? What are we going to have to do?

Alex: Make plays.

Casandra: Come up and show us how we are going to do that. Now go slow enough so the kids that need the work in making plays can see what you are doing.

At the front of the room Alex looked at the board for a moment and then picked up the checker on the 40 square.

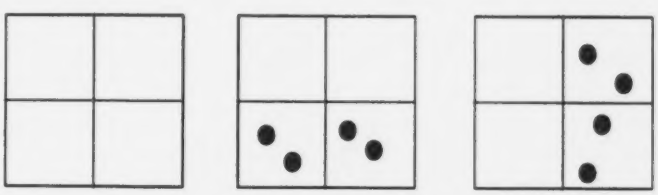
Casandra: Now what are you getting ready to do, Alex?

Alex: I took 40 and made two 20s.

Alex paused again and then turned to Casandra and asked, "Is this to make 35?"

Casandra: You're going to take 70. You want to double it so you can take half of it off.

Alex exchanged one 20 checker for two 10s while Casandra explained this play to the class. She stood right next to him, watching him look at the board. When he reached for two checkers on the 20 board she said, "No, you're fine there." She pointed to another checker and asked, "What could you do with this?" Alex seemed confused, so Jacob (who had been waving his hand vigorously in the air throughout Alex's deliberations) was invited to help. Casandra watched them very closely. She told them that whenever they got doubles (two on a square) they should put them in the corner, "so you won't bother them again." She guided them through every step to the desired representation of 2 x 35:



Then half could be removed (Observation, 4/90).

Neither Casandra nor her students discussed the meaning of halving a number or why making plays until there were exactly two checkers on each square was necessary when taking half of a number. While the text recommends that, for each play made, the student or the teacher say aloud the number sentence that goes with it (e.g., 40 is the same as two 20s), this only occurred for Alex's first few plays. The conceptual essence of the problem of halving a number seems to have gone unnoticed as Casandra marched her students through the related procedural steps to get to the right answer.

Such prescriptive or procedural treatment of conceptually oriented tasks was typical of many lessons I observed. Although the tasks presented in her teaching focused on mathematical ideas, Casandra's rule-bound treatment of them, which seemed to be drawn from her own conception of what it means to know and do mathematics, shifted their focus from exploring an idea to getting an answer. Consequently, Casandra's mathematics teaching seemed a conceptual and procedural mix.[7]

Conversations with Casandra, though, revealed little of this mix. During our interviews, Casandra stood firmly behind a view of mathematics instruction that leads students to discover and understand concepts, encourages them to figure things out, and makes mathematical ideas concrete and accessible rather than obscure and abstract. Comparing a traditional, drill approach to CSMP, she speculated:

> I can show them procedures, harp on the procedure and they will then do it. Whether or not they really understand, that I don't know. Whereas with the CSMP, they can actually see that this group of 10 is moving here by looking at the minicomputer boards. So, not only can they see the concept, but they're looking at the concept a little more concretely. (Interview, 4/90)

It is important, she feels, for a teacher to do less telling and more guiding of students to discovery, for she believes that students learn best when they have to figure things out because "when they discover things it sticks in their minds more" (Interview, 4/90). Furthermore,

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[7]This mix, or one similar, is not uncommon for teachers to exhibit when in the midst of changing their practice. See, for instance, Cohen's (1990) description of Mrs. Oublier, a teacher whose practice is influenced by state-level policy aimed at changing the way mathematics is taught in California.

mathematical problem solving requires creative solutions and students should be given space to develop their own approaches. Casandra perceives that using CSMP allows her to provide this necessary space and encouragement that she never felt as a student:

> I think that when I present it the way I do, hopefully it's . . . helping them think a little more creatively, or launching them off into their own thought processes on it. Because, I think, again, the way math was to me; it was trying to really regurgitate what the teacher is saying. (Interview, 4/90)

But Casandra's teaching doesn't always reflect these ideals. Her talk seems to be an example of what Schoenfeld (1989) calls "rhetoric of mathematical understanding." This is a way of talking about mathematics teaching and learning as being oriented toward building connections between ideas, encouraging sense making, and allowing for unconventional approaches to solving problems. According to Schoenfeld, it pervades current teaching but has not penetrated actual beliefs that are at the heart of what teachers do. It seems that Casandra has picked up the rhetoric that surrounds current reforms in mathematics education without making deeper changes in her firmly established beliefs and ways of teaching.

There were many cases, however, in which Casandra's professed beliefs were reflected in her teaching. She often asked students to explain their answers, following up students' answers with "why" questions. She sometimes encouraged her students to explore their reasoning, asking probing questions like, "Why do you think that?" and "What difference does that make?" moving completely away from the CSMP script in doing so.

How can we account for the uneven focus on meaning in Casandra's teaching? How does her use of CSMP contribute to it? And how do her beliefs influence her use of the curriculum? To understand Casandra's teaching and use of CSMP, we need to look closely at the nature of her beliefs and how they influence the choices she makes. It is important to examine the mix occurring in her teaching to determine the instances in which she is able to follow her professed beliefs as opposed to the times she betrays them. Understanding Casandra's reasoning for the decisions she makes through a close look at her teaching and beliefs may help answer these questions.

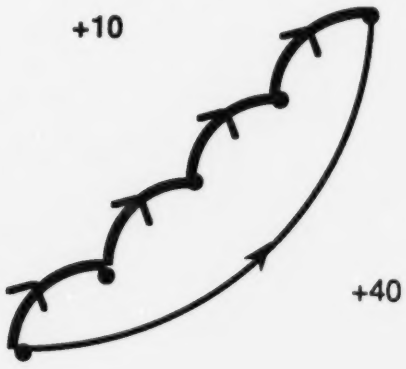
Pedagogical Reasoning and Going by the Book

Although Casandra says that she is going by the book or "sticking right with the script," she is making pedagogical decisions about her mathematics teaching continually. These vary from the role she gives CSMP in her overall curriculum to the ways in which she elaborates and modifies lessons in the manual. Even the points at which she does follow the exact script reflect pedagogical decisions. This "enacted curriculum" is, in fact, a representation of Casandra's view of mathematics as it interacts with her beliefs about learning and teaching, learners, and the context (Ball, 1988). Using these domains--content, learning and teaching, learners, and the context--I explore how Casandra's beliefs are played out in her teaching and her understanding of CSMP.

Knowing and doing mathematics: Keeping to the script. It is important to Casandra that her students understand concepts and relationships in math and use this understanding to solve problems. As she explained, "It's good for them to understand the concepts. I think they're going to be better off mathematically" (Interview, 4/90). But at the same time, she sees herself as weak in math and not capable of guiding her students toward knowing in this way. Consequently, she is highly dependent on the CSMP manual. As she sees it, using it allows her to focus on meaning in her teaching, without being limited by her lack of understanding. So she has placed her trust in CSMP and, in a sense, has abdicated her authority for knowing to its authors. And this approach seems to have paid off. She finds herself being frequently astounded by the complex thinking her students show during the CSMP lessons. "When I follow these lessons," she explained, "It's just amazing what they've picked up." Thanks to CSMP, she feels her students are way beyond her in their understanding of mathematical concepts and their ability to solve problems creatively and manipulate the CSMP representations, particularly the minicomputer.

The procedural and conceptual mix prevalent in Casandra' teaching may be partly a consequence of abdicating her authority to the text. Knowing and doing mathematics in her class has come to mean sticking to the script in the CSMP manual, providing the correct

response, and performing the procedures accurately to get to the expected end. Her reliance on CSMP has led her to implement it somewhat procedurally, to fish for correct answers or mold student responses to fit the script and suggestions in the manual. For instance, in the above minicomputer example the essential point seemed to be the procedural steps--getting two checkers on each square so half could be taken away--rather than representing what it means to take half of a number. At other times, when student responses did not follow the script, Casandra rephrased her question to elicit a specific answer. During a lesson that began with the following arrow road:[8]



Asked for the ending number if the starting number was 87, Lizzy said 47 and explained that "87 take away 40 is 47." Casandra wrote Lizzy's calculation on the board:

$$\begin{array}{r} 87 \\ \underline{-40} \\ 47 \end{array}$$

The CSMP manual actually suggests that several students be asked to tell how they did the calculation, suggesting Lizzy's solution and 87 minus 10 four times as two possibilities.

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[8]Also a pictorial representation used in CSMP, arrow roads are colored arrows connecting dots which stand for numbers or objects. Each arrow represents a different function. This arrow road is made up of four +10 functions (red) and one +40 functions (blue).

Casandra: Is there another way we could have calculated that?

Alex: 47 times 2? (This gave rise to some inaudible murmurs in the class.)

Casandra: I know what he's trying to say. Laura?

Laura: 87 times a half?

Casandra: (Looking at her manual) I should have said how would you calculate it using our arrow roads. Lizzy used the blue one. If you wanted to use the red how would you do it in your head, Amanda?

Amanda: Put 87 at the dot there and then count by ten's. Go 87, 77, 67, 57, 47.

As Amanda spoke, Casandra wrote the following on, the board:

87
-10
77
-10
67
-10
57
-10
47

Casandra: Okay, so you keep taking 10 away from where you land and you're going to come out with 47.

Amanda: Yeh. (Observation, 4/90)

It seemed as if getting the other solution suggested in the manual--87 minus 10 four times--was so important that Casandra focused on getting this from the class. She did not hear, nor choose to investigate, the possible understanding or misunderstanding embedded in the unanticipated answers given by Alex and Laura. Seeing that her original question did not elicit the book's solution she rephrased it, padding it with enough detail to elicit the response found in the manual. So it seems that Casandra's lack of mathematical understanding led her to rely heavily on the written curriculum to such an extent that she could not accept responses not in the script.

What is particularly interesting about Casandra's case is that her mathematical knowledge, from what I could tell, is not as weak as she perceives. She recognizes that she is a product of a generation that did not encourage girls in math and seems to have accepted this as her fate. But in our interviews she was able to discuss many important concepts underlying

much of the mathematics in CSMP. The mathematical errors made and accepted in class seemed to be more often a result of not attending to the mathematical content than of knowledge gaps. During interviews she was able to discuss many mathematical topics that she had glazed over in her teaching. This is not to say that Casandra's mathematical knowledge is detailed and flexible or that she views mathematics as a way of thinking. Neither does she see discourse as being part of mathematics. But there was evidence of her ability to conceptually engage the content she taught, analyze student errors, and reflect on the concepts underlying various computational skills, including regrouping to subtract, single-digit multiplication, and finding equivalent fractions. She, in fact, explained that it wasn't until she was an adult that she discovered that she "had a very good business head" and has been successful in many real-life endeavors involving mathematics. That this knowledge has not led her to taking a more active role in the mathematics she is teaching may also have to do with her beliefs about teaching and the authority of published curriculum in the classroom. It may also reflect a distinction she is making in her mind between school mathematics and its application in the real world.

Casandra seemed freer to move from the lesson scripts when the class was dealing with mathematical topics and representations with which she was fairly comfortable: The lessons that I observed on fractions and subtracting two- and three-digit numbers contained many more opportunities for students to focus on ideas and explain their thinking than the lessons that used the minicomputer to work with decimals, even if it meant veering from the lesson script. This was not surprising, for Casandra explained in an interview that she still finds the minicomputer quite complex. It is in these lessons that she seems to be especially wedded to the script and tied to the procedural steps of making correct plays on the minicomputer, molding student responses to fit the script in the manual.

But even when Casandra moved away from the scripts in the manual, there seemed to be a script that remained in her head. Although many of her elaborations of the text posed questions with potentially divergent solutions, such as "How do you know?" or "Why do you

think . . . ?" in her head they had specific answers. Repeating questions several times, Casandra often searched for the answer she had in mind. On one particular occasion the lesson revolved around a story about different numbers that had formed a +2 arrow road beginning with 1. (This particular arrow road included all odd numbers.) An arrow at the end of the road, in addition to details in the story, indicated that the string of numbers continued beyond what was pictured on the page. Casandra asked, "How do you know [that the numbers go onto the next page]?" This question was not part of the lesson script. Several students gave answers that Casandra had not anticipated, although they addressed the question in some way. She repeated each response matter-of-factly and then asked the question again. After several unsuccessful attempts at finding the desired response, she looked at Jacob, who was halfway out of his chair with his arm stretched in the air, and said, "Jacob, I think you know what I'm trying to get at." And indeed, he did. He explained that the arrows at the end of the arrow road indicated that "It could go on and on." "That's right," Casandra affirmed and continued with the next question in the lesson script.

Casandra's tendency to shape the classroom discourse so that it followed a script seemed to be more than a way of dealing with domains of mathematics with which she was uncomfortable, for it pervaded each domain of her math teaching. In the above instance, the script was her own, added to the CSMP lesson, suggesting specific understandings she felt her students should have. That the additions Casandra makes are in "script" form, having very specific answers, implies a convergent view of the nature of mathematics and mathematical knowing: that there are single, right answers and knowing means giving these answers. Such a view of mathematics is compatible with the use of a scripted lesson format.

So in one sense, the scripted format of CSMP supports her beliefs about the nature of mathematics, although the content of the scripted lessons--the emphasis on divergent strategies and solutions--opposes this view. Casandra interprets the possible student response in the CSMP manual as the right answer. Her lessons, then, are series of questions and answers, rather than whole-class explorations of a problem. Reflecting back to Casandra's description

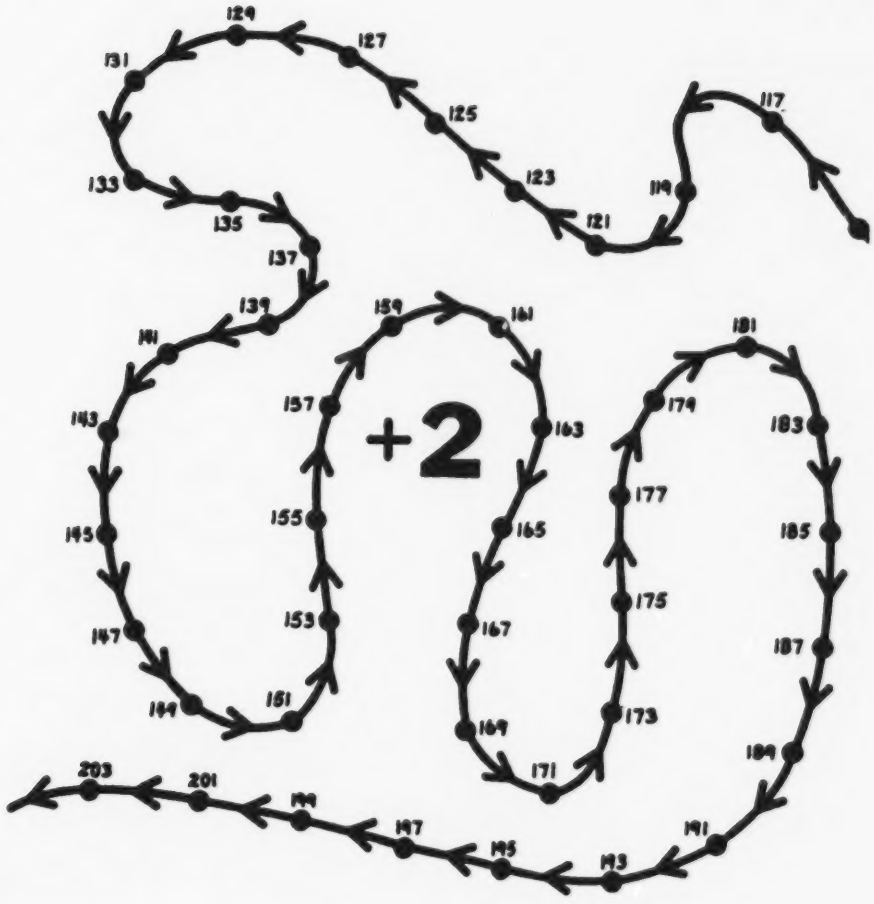

Figure 1. This +2 arrow road begins with a dot labeled "1" and ends with and arrow after the dot labeled "203."

of her mathematics classes in high school--in which the goal was to regurgitate what the teacher said, "to stay right on line with that, so I'm not wrong" (Interview, 4/90)--helps explain the emphasis on right answers in her teaching, although her interviews reflect a desire to encourage divergent thinking. While the content of CSMP has challenged Casandra to expand her view of mathematical knowing through the kinds of questions asked, the scripted lesson format has only reaffirmed her traditional views about the nature of mathematics and knowing as having the right answer.

Contributing to this conflict between divergent and convergent discourse in Casandra's teaching are her beliefs about learners, learning and teaching, and the context. As proposed above, how Casandra thinks about knowing mathematics influences how she shapes the classroom discourse and how she listens to students. Further influencing her practice are her beliefs about learners and learning. These, however, are also influenced by her use of CSMP and continue to change.

Knowledge of learners: Selective hearing. CSMP is a program that encourages and uses student contributions in each lesson. The student-teacher dialogue format reflects the authors' assertions that learning mathematics occurs through engaging in mathematical thinking, developing strategies to solve problems, and communicating about mathematical ideas within the context of whole-class lessons. And indeed, each of Casandra's lessons provided many opportunities for students to give solutions, explanations, and ideas. But the actual degree to which the students' ideas contributed to or were used in these lessons is unclear, for Casandra's listening seemed often to be selective. On several occasions, Casandra took no note of incorrect student responses but pressed on with the lesson script as if they made sense. For example, 240 was accepted as one of the numbers in the series of numbers 1, 3, 5 . . . (the +2 arrow road beginning with 1). Later, Casandra accepted Alex's response that $\frac{1}{512}$ would be half way between 0 and $\frac{1}{1024}$. On other occasions, she rephrased what her students said to match what she was expecting to hear. During the lesson that involved halving using the minicomputer,

the class was taking half of seven dollars. Casandra put a dot between the one's board and the board to the right of it. She then asked,

Casandra: What's the dot for?

Adam: So there's the cents on one side and the dollar on the other.

Casandra: What if I were not talking about money? What could that represent?

Stevie: (inaudible)

Casandra: You're getting closer.

Alex: The dividing point.

Casandra: It is sort of like a dividing point.

Corinna: Decimal point.

Casandra: What happens after the decimal point? What do the numbers start doing?

Amanda: They start getting fractions.

Casandra: That's right. They start getting smaller. They're smaller than one.

Casandra rephrased Amanda's reference to fractions to be saying that the numbers to the right of the decimal point are smaller than one. Whether or not this is what Amanda had in mind, Casandra's interpretation allowed the class to continue the lesson.

In spite of this selective hearing, Casandra does attend to much of what her students know in other ways. She attributed many of her decisions about her use of particulars in CSMP to what she knew about her class. When asked if she planned to approach certain elements differently next year, she stressed that she could not make those decisions until she knew that class, although she did know how she would handle it if she could go through again with the same class. Casandra also thoughtfully responded to questions I posed about individual student work, diagnosing errors and commenting on what the child seemed to understand. These, however, were hypothetical situations presented in interview contexts. Why does Casandra seem to miss these in the classroom context?

Recalling that Casandra's teaching is driven by the script in the textbook or in her head can provide one possible explanation for her selective hearing. In her quest for a specific

answer, she misses the information about her learners' knowledge embedded in their responses and passes up the opportunity to explore or build on it. Her attempt to teach mathematics for understanding by following CSMP has led her to stick to the script and, ironically, draws her away from hearing what her learners understand.

That Casandra is not inclined to hear her students' incorrect or partially correct answers or interpret them as being important or useful may also reflect her view of mathematics as consisting of single right answers. Furthermore, her seeming choice not to engage the mathematical content in the CSMP lessons but to treat it procedurally limits what she is able to hear in her students' words. Casandra's beliefs about teaching and learning may also contribute an explanation of her selective listening to students.

Learning and teaching mathematics: Doing and telling. As Casandra sees it, there are two ways of thinking about teaching, and these two are opposites; they are grounded in opposing views of how children learn. On one extreme is the theory that children learn by being told information, on the other is the perspective that children learn, not by hearing, but by doing. It is this latter view which Casandra holds. Children, she believes, "learn best" when they are "actively involved in the lesson" (Interview, 4/90). By actively involved, Casandra means actually doing something with their hands. This is what she calls "making it concrete" so it is accessible to students. Good teaching, then, includes "guid[ing] them in a way where they would be doing their own self discovering" (Interview, 4/90). Likewise, teaching that depends on telling and drilling is likely to be unsuccessful because it is too abstract and is counter to the way children naturally learn. Casandra's bifurcation of the ways that people think about teaching and learning--(a) teaching as telling and learning as listening being abstract and (b) teaching as guiding students to discovery being concrete--complicates her interpretation and use of CSMP, which seems to include both these opposing approaches.

Although Casandra interprets CSMP as guiding students to understanding through work with mathematical concepts, she also characterizes the whole-class lesson format as being "teacher centered" or "lecture." She feels that initially it may be too abstract for students.

These are two concerns that she feels the authors of CSMP did not take into account, and, as she sees it, it is because of these two oversights that she sometimes "looses the class" (Interview, 4/90). Casandra compensates for these oversights by altering pieces of the lessons, making accommodations that fit her beliefs about learning.

Feeling that students need to have less time "sitting and listening" and more opportunities to actually work with concrete manipulatives, Casandra makes occasional additions to the CSMP lessons to "get them more involved" (Interview, 4/90). One lesson, for example, on equivalent fractions was designed so that the class would be dividing and shading in parts of circles on the board to demonstrate equivalency. Feeling that the students needed to actually do their own dividing and shading to really understand, Casandra had each student divide and shade his or her own circles on paper. This minor change in the lesson, she explained, made the concept more concrete.

This was not the only time that Casandra found the whole-class, "teacher-at-the-front format" problematic (Interview, 4/90). In general, she worries about keeping the students engaged in the lessons and monitoring their progress. Individual worksheets or practice pages have accomplished both these objectives in the past. She is also concerned that the lesson accommodations that she introduces might undermine the spirit of CSMP. She frequently applies various teacher effectiveness techniques to encourage student participation. Some of these include calling many students to the board to write or draw their solutions and drawing students' names from a basket of sticks, rather than calling on those with raised hands. These techniques, she explained, "raise their level of concern" (Interview, 4/90) and encourage them to pay attention. While she feels uncomfortable using these approaches, she sees no other choice; student participation, in her eyes, is necessary for their learning and so she can "evaluate what they're doing" (Observation, 4/90):

> If you allow it, the kids that know it will be the kids that are the most responsive, and there is that technique that I use from effective teaching to try to get a response from everyone. I think sometimes that is intimidating, especially for kids that want to sort of sink back and hide and not respond. So, I use that stick method, where they're randomly called on. . . . They could sit

> back and I could let them sit back and it would seem just wonderful to have all these wonderful responses, but they will be coming from the same people. . . . Then they soon turn off and feel they don't really have to. Somebody else will know it or answer it for them. (Interview, 4/90)

What Casandra finds particularly ironic is that she feels she has to use an approach that can be intimidating to some students in conjunction with CSMP, a program, as she sees it, that is designed to draw students into mathematics, rather than intimidate them.

By using effective teaching techniques in conjunction with CSMP, Casandra reconciles the conflicts between her beliefs about learning and teaching with those she assumes the program authors to hold. What is seen in the classroom is an interesting mix of both approaches mediated by Casandra's beliefs and assumptions about what it means to learn, know, and do mathematics. These beliefs and assumptions also interact with Casandra's views about the contexts in which she teaches.

The context: Nested situations. Casandra teaches CSMP in several contexts that are nested in one another: CSMP is part of her total math curriculum, which is only part of her complete curriculum. Her classroom is located in Valleyrock Elementary School, which has its home in a larger community and school district. But this community also sits within a larger society, or what she calls the "real world." The most salient of these having an impact on Casandra's decisions about CSMP may be the affluent community Valleyrock serves. Knowing that her students' parents are skeptical of CSMP and are concerned that their children master basic skills typically emphasized in traditional approaches to teaching mathematics, Casandra provides her students with daily computational practice from a traditional textbook. At the same time, demands from the district and educational community push for innovation in teaching and the use of CSMP as the core curriculum. But all of these contextual demands influence the choices Casandra makes. Both explicitly and implicitly, Casandra's use of CSMP is shaped by the messages she perceives as defining her role as teacher.

The contexts that influence teaching from inside the classroom are more subtle. Casandra teaches many different subjects. How she manages them and the time and significance they are allotted reflect pedagogical decisions that are shaped by her beliefs

about learning, learners, the content, and the demands of the context. Considering how her beliefs vary in different contexts within the classroom helps to explain the mix we see in Casandra's use of CSMP. For example, Casandra has bifurcated her math teaching into two parts: "computational" and "conceptual." Each has its own purpose and tasks associated with it. She sees whole-class lessons as addressing the conceptual elements of mathematics and the Addison-Wesley textbook pages as developing computational mastery. While she claims that the key to understanding, enjoying, and excelling in mathematics is found in CSMP's conceptual approach, students also need to know how to compute. She describes this as a political decision, "because I know my parents," but she also admits that she prefers to use both and depends on each to fulfill its purpose. After all, they need both computational ability and conceptual understanding in the "real world" (Interview, 4/90).

Although Casandra sees herself as treating the two types of math quite separately--computation in the morning, CSMP in the afternoon--there are times when the two overlap or the distinctions between them become less clear. I observed a possible overlap in the way Casandra and her students treated the workbook pages that are part of some CSMP lessons.[9] Casandra assigned all of her students all four of the workbook pages related to the lesson. These involved putting decimal numbers on the minicomputer and reading numbers on the boards. After most of the students had completed them, she announced that it was time to check them. Red pencils were hurriedly obtained. Casandra quickly read aloud the answer to each problem or had a student put it on the board while the students checked their papers. Unlike the lesson that had just been completed, no explanations were given for the answers and errors were not discussed. There were several instances in which many children had wrong answers and seemed confused, but Casandra kept going. "What if we didn't know?" one child

[9]These three to four pages are designed to reinforce and extend the concepts developed in the lesson and are quite unlike traditional textbook pages in content and format. As each page increases in difficulty, the authors recommend that not all students be expected to do all pages.

complained when she found that she had incorrectly put 9.3 on the minicomputer. "Well, you have to circle it; it's wrong," Casandra responded.

This emphasis on right answers in total isolation from their meaning was unlike any of the whole-class lessons I observed. It was as if the workbook pages, a task more often associated with traditional ways of doing math, had caused the class to revert to emphasizing only right answers devoid of any meaning. In a later conversation with Casandra, I learned that, from behind her desk, she had not noticed much of the confusion over these pages. She diagnosed their errors as a result of "not paying attention" and rushing through the pages, rather than a lack of understanding. It seems that, in this situation, ways of knowing and doing math, which are usually restricted to traditional math, encroached on CSMP. The context determined by the task--doing and correcting workbook pages--dictated what it meant to do math.

Subtle beliefs such as these are deeply rooted and developed over many years of experience in schools (Lortie, 1975). They are wrapped up in the teacher's beliefs about the purposes of schools and learning of particular content (Ball, 1988) and are necessarily intertwined with beliefs about the substance and nature of the content, teaching and learning, and learners. Together they are powerful determinants in how a teacher like Casandra understands and uses a curriculum like CSMP.

Empowering Students

My observations also revealed an additional factor, other than Casandra's beliefs, that shaped her use of CSMP: the students. This seems to be a consequence of the disciplinary perspective of mathematics proposed by CSMP, in which classroom discourse is an essential part of the mathematics and the authors' choice to use a scripted format in suggesting how the lessons might proceed. Teaching mathematics from this perspective necessarily involves opening the floor of the classroom to student ideas and thoughts, which is likely to move the discourse away from the lesson scripts, for students don't always respond in the way the authors predict. It invites student ideas and questions to reconstruct the lesson.

This became problematic in many of Casandra's lessons as student responses and questions pulled her away from the script and required that she create her own. In doing so, or in turning the students back to the script, Casandra relied on her knowledge of the mathematics and beliefs about learning it. Just as following the text led Casandra to presenting a mixture of conceptually and procedurally oriented instruction, it also opened the door to student-initiated steps away from the lesson script. In other words, by abdicating her authority for knowing to CSMP and following the script, Casandra actually created more uncertainty for herself because the script opens the floor to student discourse. The role of students in shaping what happens in the classroom and teachers' response both need to be explored more as we advocate mathematics teaching that is grounded in the discipline and necessarily includes student discourse. Casandra looked to the text as the authority. Nevertheless, the nature of mathematics teaching and learning implicit in the CSMP curriculum passed this authority back to her.

Conclusion

In Casandra Singer we see a teacher who is consciously trying to change her mathematics teaching through the use of an innovative curriculum. Wanting to provide her students with an opportunity to understand mathematics but feeling limited by her own knowledge, she relies on the lesson scripts. This way she can allow her students to go beyond her understanding. But "following the book" has resulted in a mixture of procedurally and conceptually oriented teaching, as she tries to push her students to conform to the script either in the manual or her head. Her trust of CSMP has lead her to abdicate her own authority for knowing as a teacher to the authors of the program. She has also relinquished responsibility for engaging the mathematical content of the lessons and of what her students say. Yet the nature of CSMP, with its very emphasis on whole-class exploration of mathematical ideas, increases her responsibility for exploring these ideas herself--a task she feels unable to accept.

The case of Casandra Singer raises questions about the delicate balance of authority between the curriculum and the teacher and students. CSMP is a carefully designed curriculum

that has the potential to revolutionize one's teaching. But CSMP does not manage the mathematics for the teacher. Indeed, it invites the teacher to engage mathematical problems with her students in ways that traditional texts and practices do not. This invitation assumes certain knowledge of and dispositions toward mathematics. Lacking what she feels is an appropriate level of mathematical knowledge, Casandra relinquishes her authority as a teacher to the experts behind the script. This in turn limits her use of the program, and inhibits her from listening to her students.

Cassandra's use of CSMP raises more issues than that of teachers' subject-matter knowledge. Her procedural use of CSMP reflects the relationship she has constructed between the teacher and the textbook and the power with which she has invested the published curriculum. The question that Casandra's case leaves unanswered is how to help teachers use curricula as resources rather than formulas. Casandra's struggles to change her mathematics teaching are twofold: She must not only learn to teach mathematics in a way that she has never experienced but she must learn to use curriculum in a new way as well. This requires accepting, rather than abdicating, authority for knowing mathematics.

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